34.6 **$70°F$ atmospheric air enters the compressor of an air-standard gas turbine and is compressed at a 4:1 ratio. The air is $1200°F$ when it enters the turbine and expands to $1atm$. The compressor and turbine efficiencies are 75% and 85%, respectively. What is the thermal efficiency of the cycle?**

A. 15%

B. 23%

C. 36%

D. 45%

A gas turbine consists of a compressor, combustor, and turbine as depicted in the reference handbook under Brayton Cycle. Consider the air entering the compressor as State 1, the air leaving the compressor and entering the combustor as State 2, the air leaving the combustor and entering the turbine as State 3, and the air leaving the turbine as State 4. For an air-standard cycle, it is valid to use constant, ideal, specific heat capacities and work with the temperature values rather than enthalpy. Use the equation for cycle efficiency. Factor out and cancel c_p.

$$\eta_{cycle} = \frac{\dot{W}_{net}}{\dot{Q}_{net}} = \frac{\dot{W}_{12} + \dot{W}_{34}}{\dot{Q}_{23}} = \frac{c_p\left(T_1 - T_2^{'}\right) + c_p\left(T_3 - T_4^{'}\right)}{c_p(T_3 - T_2^{'})} = \frac{\left(T_1 - T_2^{'}\right) + \left(T_3 - T_4^{'}\right)}{(T_3 - T_2^{'})}$$

The temperatures at State 1 and State 3 are given. The temperatures at State 2 and State 4 can be determined by initially assuming isentropic compression/expansion, then adjusting for the efficiency of the compressor and turbine. Starting with the compressor, apply the equation for a Constant Entropy Process to find the *ideal* compressor exit temperature. Use absolute temperatures i.e. Rankine and convert back to Farenheit at the end.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\left(\frac{k-1}{k}\right)}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = (530R)\,(4)^{\frac{1.4-1}{1.4}} = 787.6R = 327.6°F$$

Apply the compressor efficiency to find the *actual* compressor exit temperature.

$$\eta_C = \frac{T_{es} - T_i}{T_e - T_i} = \frac{T_2 - T_1}{T_2^{'} - T_1}$$

$$T_2^{'} = T_1 + \frac{(T_2 - T_1)}{\eta_C}$$

$$T_2^{'} = T_1 + \frac{(T_2 - T_1)}{\eta_C} = 70°F + \frac{(327.6°F - 70°F)}{0.75} = 413.5°F$$

Determine T_4, again assuming a constant entropy process. Note $P_3 = P_2$ because the heating in the combustor is considered to be a constant pressure process. The turbine discharges to atmospheric pressure, therefore the expansion ratio of the turbine is the inverse of the compression ratio of the compressor.

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\left(\frac{k-1}{k}\right)}$$

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = (1660R)\,(0.25)^{\frac{1.4-1}{1.4}} = 1117.1R = 657.1^{\circ}F$$

Apply the turbine efficiency to find the *actual* turbine exit temperature.

$$\eta_{turbine} = \frac{T_3 - T_4^{'}}{T_3 - T_4}$$

$$T_4^{'} = T_3 - \eta_{turbine}\,(T_3 - T_4) = 1200^{\circ}F - (0.85)\,(1200^{\circ}F - 657.1^{\circ}F) = 738.5^{\circ}F$$

Substitute temperatures into the original equation for the efficiency of the cycle. Notice the compressor term turns out to be negative because work is being done *on the cycle*, whereas the turbine term is positive because work is being done *by the cycle*. The numerator is the net work produced by the cycle.

$$\eta_{cycle} = \frac{(70^{\circ}F - 413.5^{\circ}F) + (1200^{\circ}F - 738.5^{\circ}F)}{(1200^{\circ}F - 413.5^{\circ}F)} = 0.15$$

Answer A